lect 2

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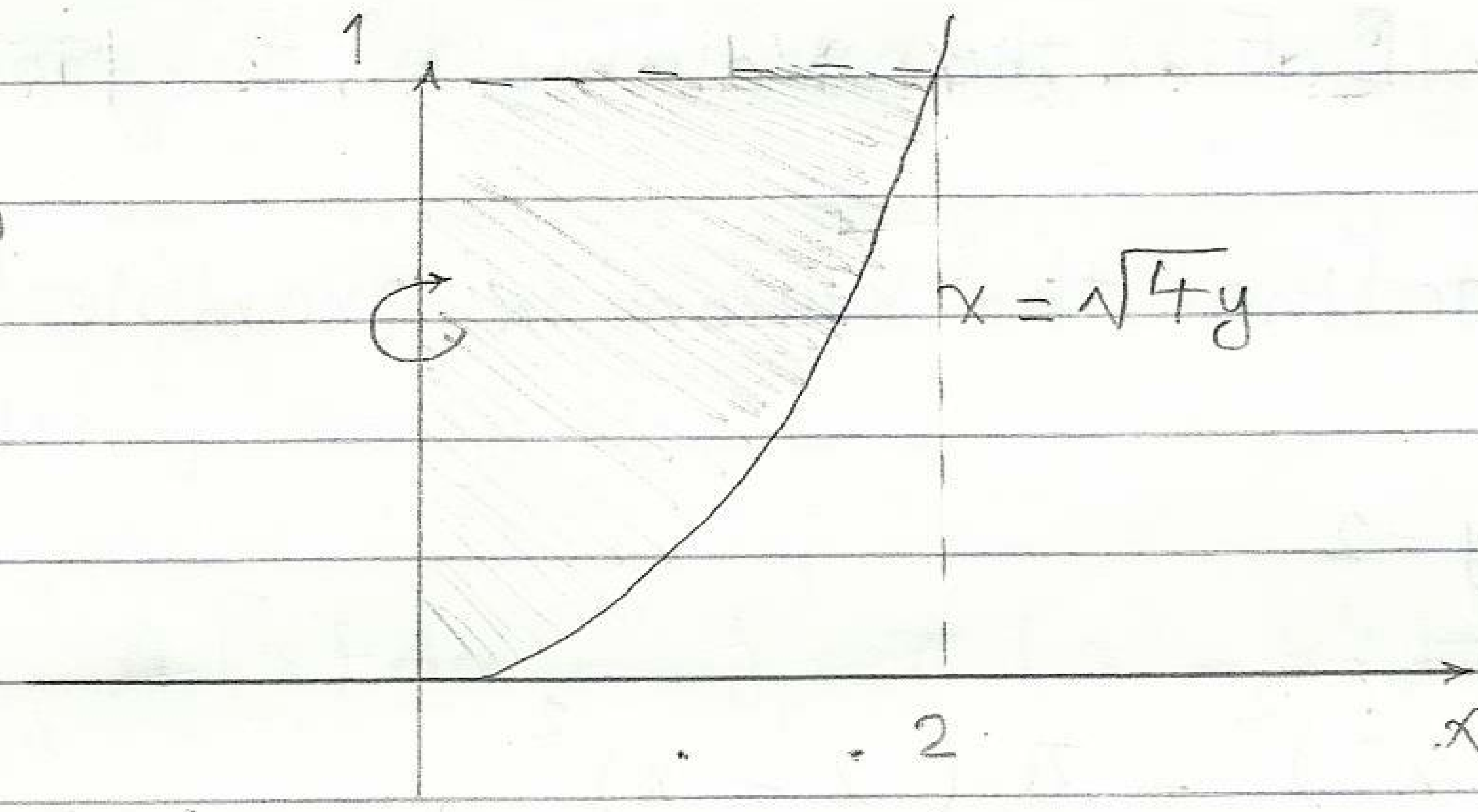
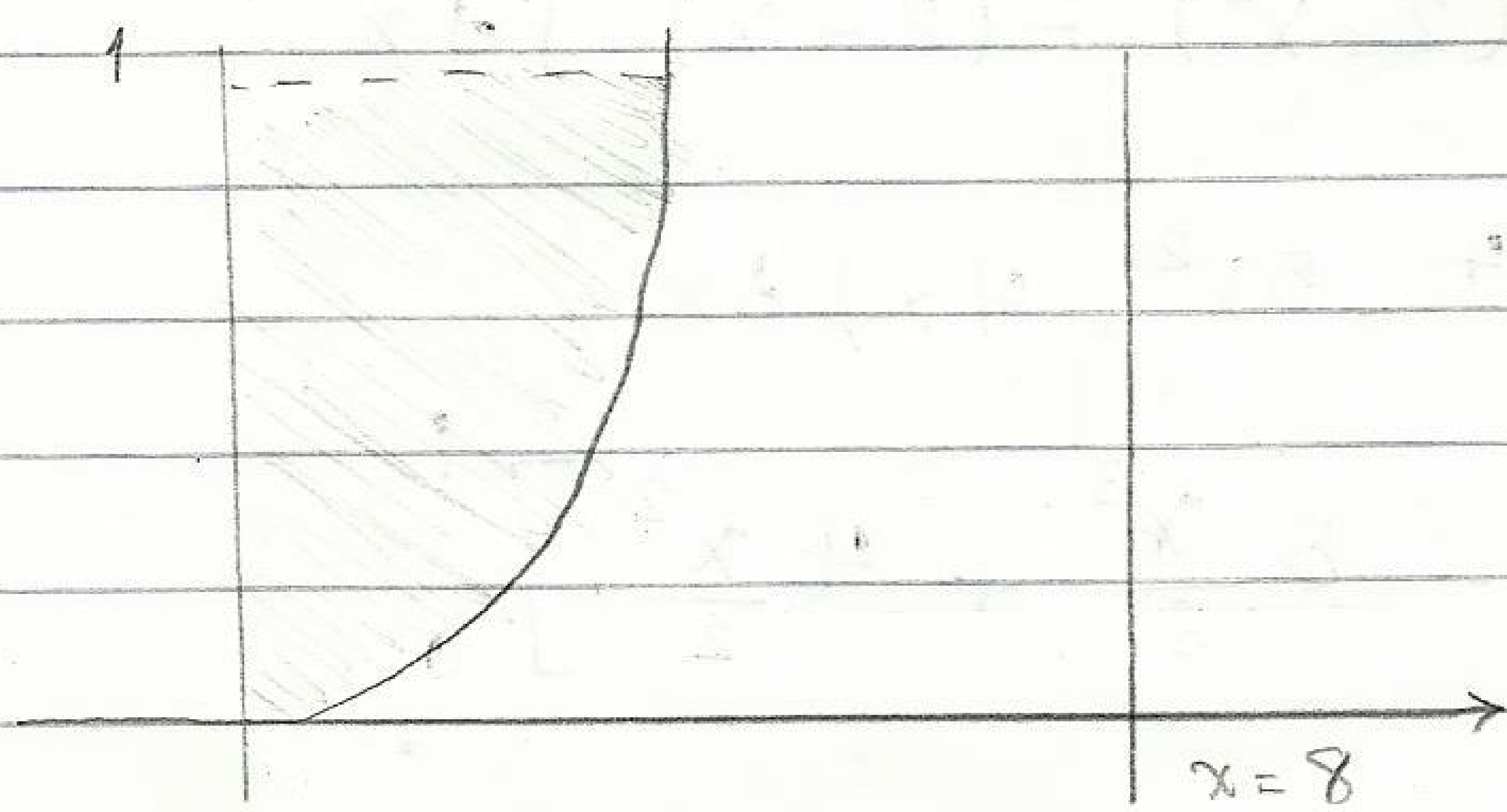
$x = \sqrt{4y}$



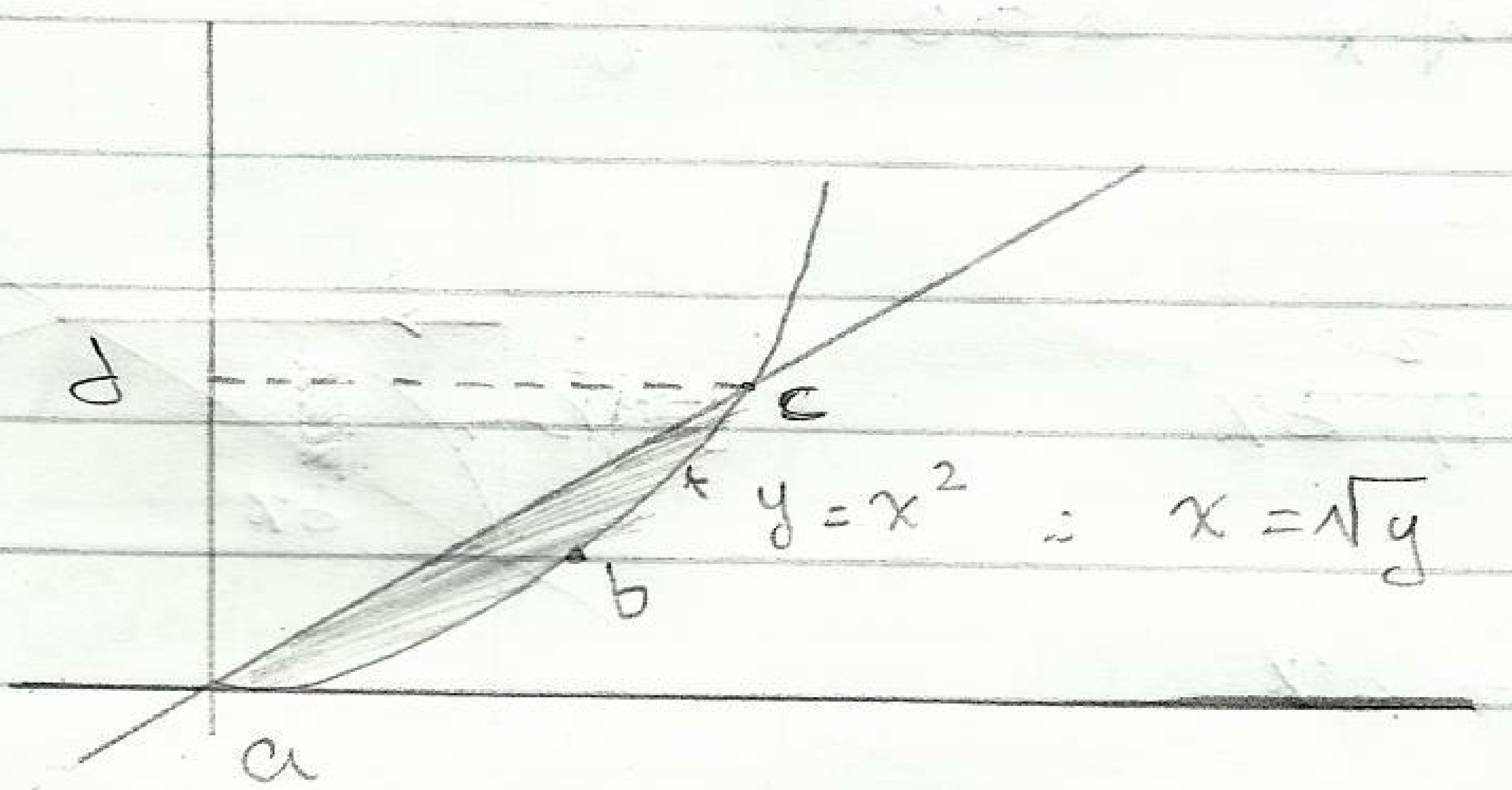
$$V = \pi \int_0^1 [\sqrt{4y}]^2 \, dy$$



$$V_{cylinder} = \pi (8)^2 (1)$$

$$V_1 = \pi \int_0^1 [8 - \sqrt{4y}]^2 \, dy$$

6.3 Volumes By Cylinder Shells



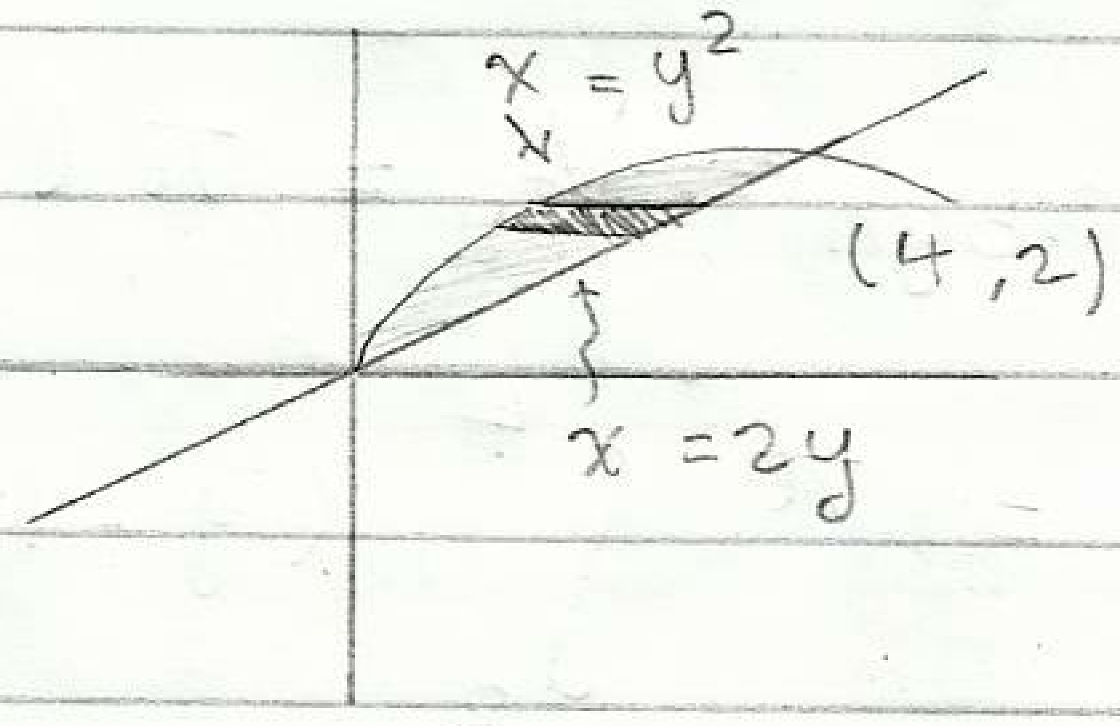
$$V_{abcd} = \pi \int_0^1 (\sqrt{y})^2 \, dy$$

$$V_{acd} = \frac{1}{3}\pi (1)^2 (1) \quad \text{«Cone»}$$
$$= \frac{1}{3}\pi$$

NoTe $\quad |\tan x|^4 = \tan^4 x$

6.2 $\boxed{9}$ $y^2 = x \qquad x = 2y \qquad$ about y-axis

$y = \sqrt{x} \qquad y = \frac{1}{2}x$



$y^2 = 2y \quad , \; y = 2 \quad x = 4$

$\quad , \; y = 0 \quad x = 0$

$$V = \pi \int_0^2 \left[(2y)^2 - (y^2)^2\right] dy$$

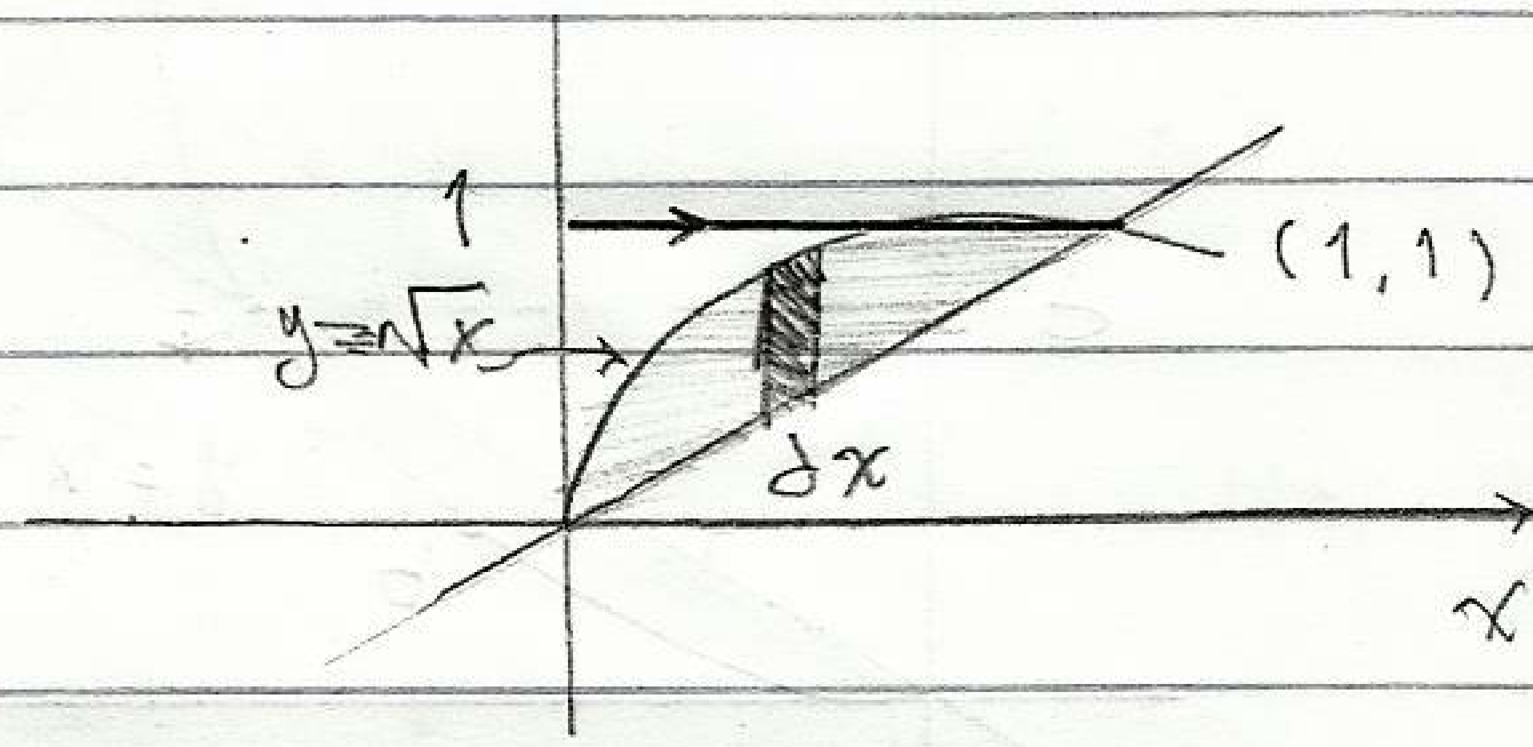
$$= \pi \int_0^2 [4y^2 - y^4] \, dy = \pi \left[\frac{4}{3}y^3 - \frac{1}{5}y^5\right]_0^2$$

$$V = \pi\left[\frac{4}{3}(2)^3 - \frac{1}{5}(2)^5\right] = \frac{64\pi}{15}$$

$\boxed{11}$ $y = x$, $y = \sqrt{x}$ about $y = 1$

$$V = \pi\int_0^1 (\sqrt{x}-1)^2 - (x-1)^2 \, dx$$

$$= x - 2\sqrt{x} - x^2 + 2x$$



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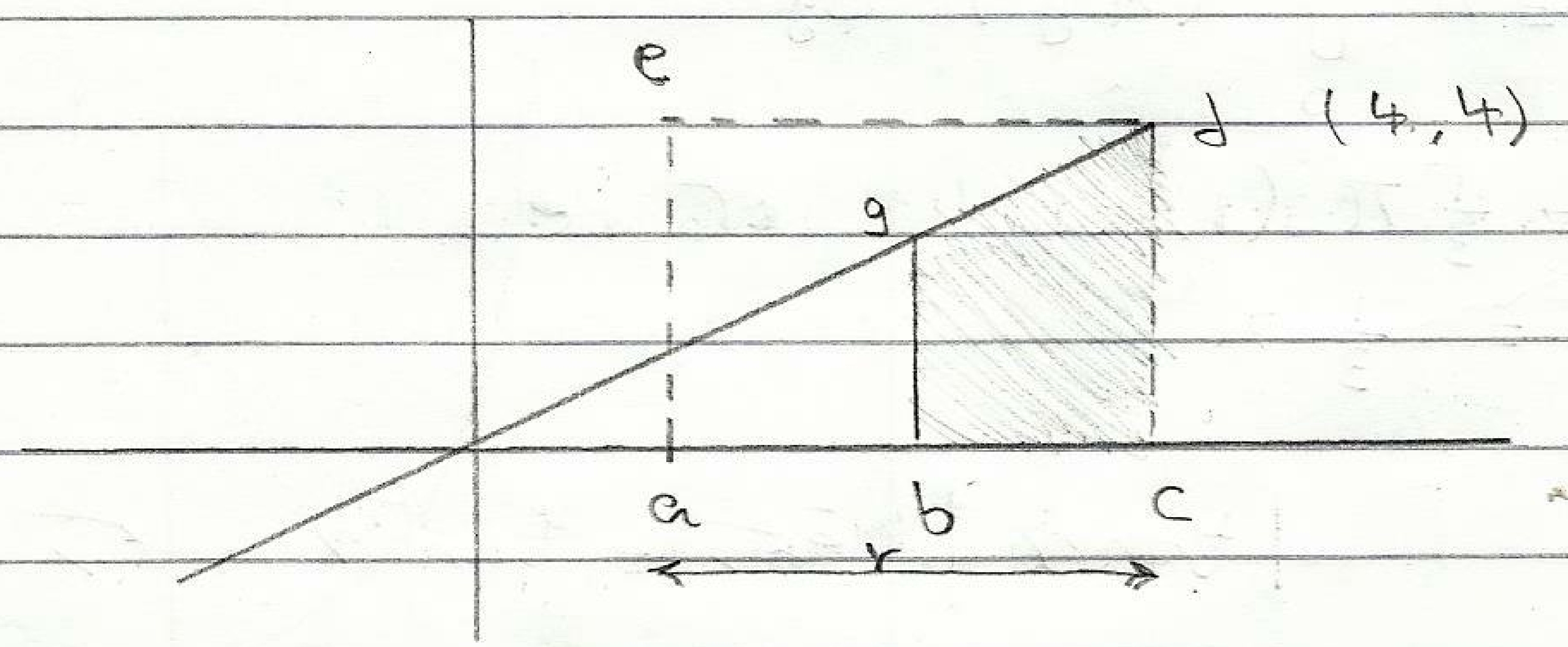


$$V_{abcde} = V_{cy} \cdot \pi_4 (3)^2 (4)$$

$$V_{abgh} = V_{cy} = \pi (1)^2 (2)$$

$$V = \pi \int_2^4 (y-1)^2 \, dy$$

6.5 Average Value of a Function

A) Discret f :-

$$y_{Av} = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

B) Cont $f'(s)$

$$f_{av} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

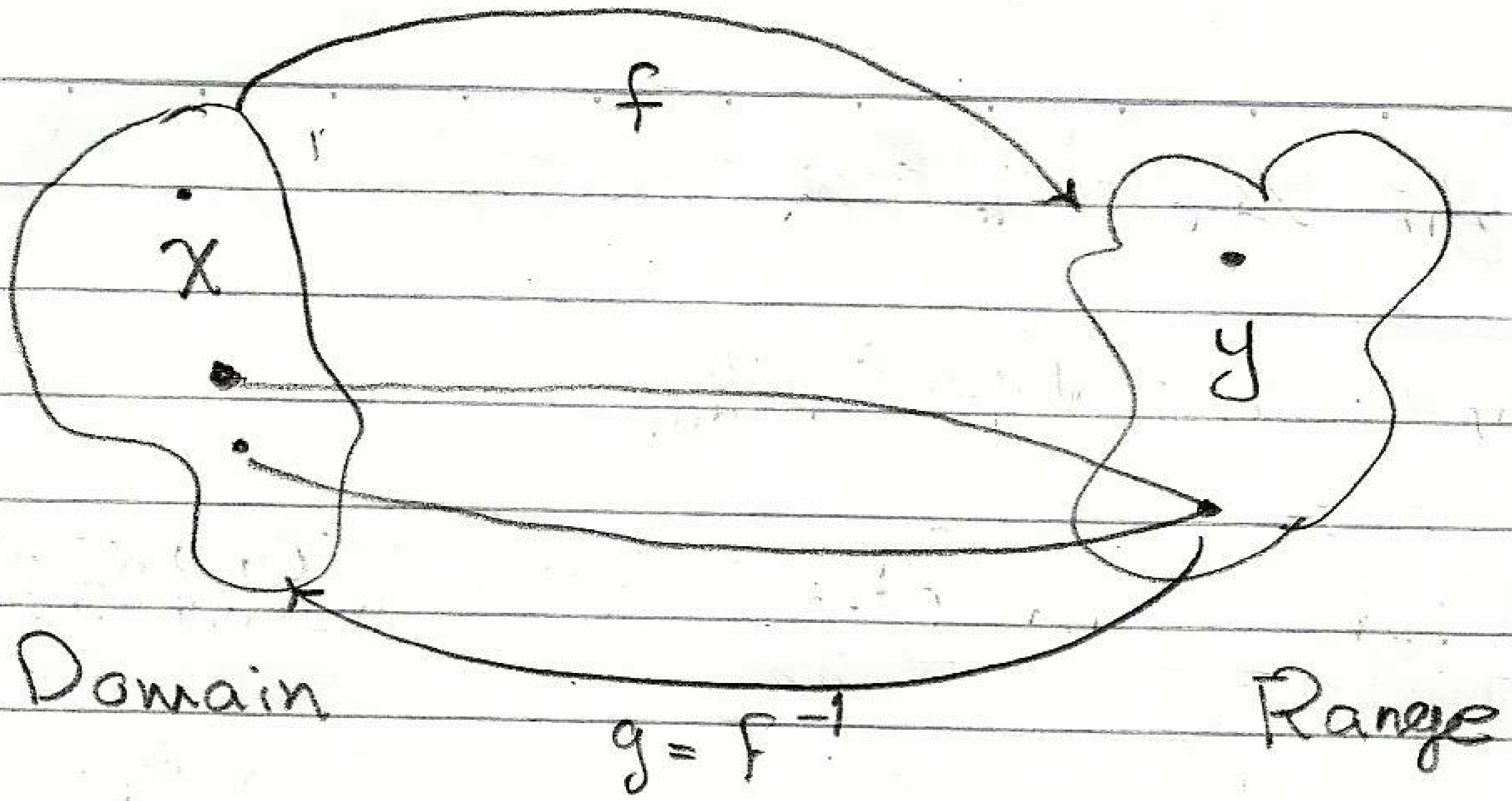
- The mean Value theorem for Integers :-

If $f(i)$ Continuous on the Closer Integer $[a,b]$
عم ذكرها اشاغل

, then the exist at least a number c in $[a,b]$ such that $f(c) = \frac{1}{b-a} \int_b^a f(x)\, dx$

Since $f(x) = 1 + x^2$ is Polynomial so its Cont (theorem)

CH 7 Inverse Functions



$(A \times B) = C \times B$

ده معناه ان $C = A$ لازم لكن لـ Matrices خطأ «False»

T or F Q

$K \times a = 0$ لا يعني انه واحد منهم لازم يساوي الصفر في الـ Matrices

$\boxed{1}$ $\text{Dom } f^{-1} = \text{Range } f$

$\boxed{2}$ $\text{Range } f^{-1} = \text{Dom } f$

$\boxed{3}$ $f^{-1}(x) \neq \dfrac{1}{f(x)}$

$\boxed{4}$ $f^{-1}(f(-x)) = x = f(f^{-1}(x))$

Ex: Let $f(x) = x^3$, $g(x) = x^{\frac{1}{3}}$

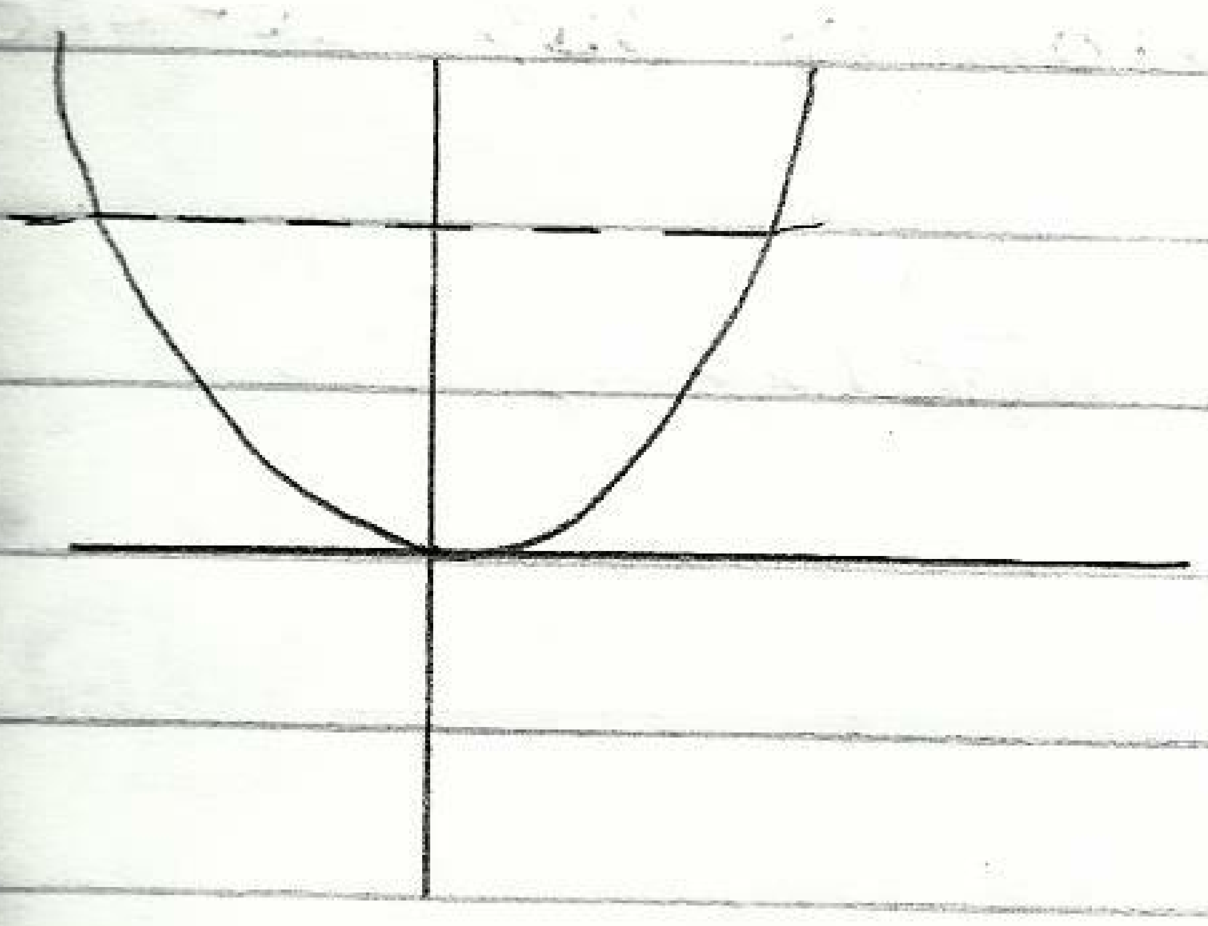
$f \circ g = f(g(x)) = f(x) = (x^{\frac{1}{3}})^3 = x$

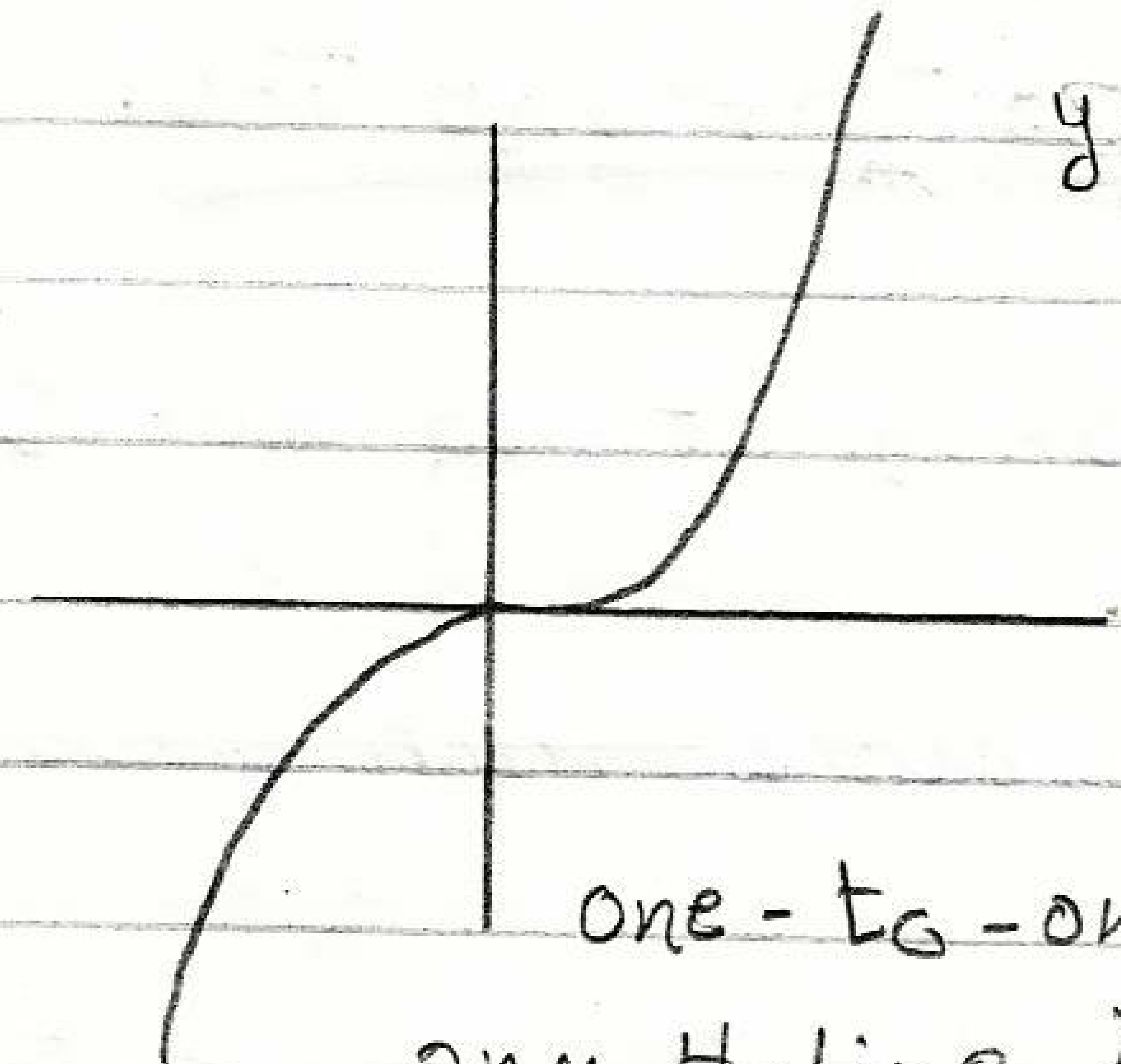
$g\circ f = g(f(x)) = g(x^3)^{\frac{1}{3}} = x$

g is the inv find f vice verse

Ex 2 • $\frac{d}{dx}\tan^{-1}(\tan x^3) = 3x^2$ ((Important 4 exam))

• $\frac{d}{dx}\sin^{-1}(3\sin x^2) \neq$ لا تحل بسبب الرقم 3





$y = x^3$

$x = 2 \rightarrow y = 4$

$x = -4 \rightarrow y = 4$

one-to-one since any H-line int in only one pt

• It has no inverse since $\begin{cases} x = \pm 2 \Rightarrow y = 4 \\ \text{any horizontal line intersection in more than 1 point} \end{cases}$

• **Theorem** :- Every Inc/dec functions are one-to-one

– $y = 5x + \cos x$ $\quad y' = 5 - \sin x > 0 \ \forall x$

for all x

DATE

1) **Definition** :- A function f is called a one to one function if it never takes on the same value twice; that is :- $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

- **Horizontal line test** :- A function is one-to-one if & only if no horizontal line intersects its graph more than once.

- $f^{-1}(y) = x \iff f(x) = y$

★ **How to find the Inverse function f^{-1}**

Step 1 :- Write $y = f(x)$

Step 2 Solve this equation for x in terms of y (if possible)

Step 3 To express f^{-1} as a function of x, Interchange x & y. The resulting equation is

$y = f^{-1}(x)$

Ex $f(x) = x^3 + 2$, $f'(x) = 3x^2$ « inc »

$y = x^3 + 2$, $x^3 = y - 2$, $x = (y-2)^{\frac{1}{3}}$

$f^{-1}(x) = (x-2)^{\frac{1}{3}}$

Check $f(f^{-1}(x)) = f(x-2)^{\frac{1}{3}}$

$$= \left[(x-2)^{\frac{1}{3}}\right] + 2 = x$$

Range : R

The graph of f^{-1} is obtained by reflecting the graph of f about the line $y = x$

Ex Find the inverse function

$f(x) = \sqrt{x+2}$ & Sketch f^{-1}

Solution $y = \sqrt{x+2}$

$y^2 = x+2 \longrightarrow x = y^2 - 2 \longrightarrow y = x^2 - 2$

$f^{-1}(x) = x^2 - 2$